Fresh look at the Hagedorn mass spectrum as seen in the experiments

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The medium dependent finite width is introduced into an exactly solvable model with the general mass-volume spectrum of the QGP bags. The model allows us to estimate the minimal value of the QGP bags' width from the lattice QCD data. The large width of the QGP bags not only explains the observed deficit in the number of hadronic resonances comparing to the Hagedorn mass spectrum, but also clarifies the reason why the heavy QGP bags cannot be directly observed as metastable states in a hadronic phase.

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I. INTRODUCTION AND PUTTING FORWARD THE PROBLEM

The statistical bootstrap model (SBM) [1] was the first to suggest that the exponentially increasing mass spectrum of hadrons, the Hagedorn spectrum, could lead to new thermodynamics above the Hagedorn temperature T_H. Shortly after, it has been demonstrated that both the dual resonance model (DRM) [2, 3] (which originated the string-like picture of hadrons) and the MIT bag model (which supposes the nontrivial vacuum structure) resemble the other features of SBM besides the asymptotic form of mass spectrum [4]. Moreover, it has been realized that the Hagedorn temperature might be interpreted as the temperature of phase transition to the partonic degrees of freedom [5]. Henceforth these results initiated the extensive study of hadron thermodynamics within the model of a gas of bags (GBM) [6]. The analytical solution of GBM with a non-zero proper volume of hadronic bags (with the hard core repulsion) allowed one to become aware of possible mechanism of deconfining phase transition from hadronic matter to the quark gluon plasma (QGP) (set by an infinite bag containing free quarks and gluons) [7]. Amazingly, up to now GBM remains one of the most efficient phenomenological instruments to successfully describe the bulk properties of hadron production in existing experimental data on relativistic heavy ion collisions and due to the simplicity of its foundations to easily incorporate newly discovered features of strongly interacting matter [8]. Apparently, the most recent attempts to update GBM bringing the contemporary knowledge of the phase diagram of quantum chromodynamics (QCD) [9, 10, 11] are entirely based on the lattice approach to quantum chromodynamics (LQCD) [12, 13].

However, despite the considerable success of these models and their remarkable features all of them face two conceptual difficulties. The first one can be formulated by asking a very simple question: 'Why are the QGP bags never directly observed in the experiments?' The routine argument applied to both high energy heavy ion and hadron collisions is that there exists a phase transition and, hence, the huge energy gap separating the QGP bags from the ordinary (light) hadrons prevents the QGP co-existence with the hadrons at densities below the phase transition. The same line of arguments is also valid if the strong cross-over exists. But on the other hand in the laboratory experiments we are dealing with the finite systems and it is known from the exact analytical solutions of the constrained statistical multifragmentation model (SMM) [14] and GBM [9] that there is a non-negligible probability to find the small and not too heavy QGP bags in thermally equilibrated finite systems even in the confined (hadronic) phase. Therefore, for finite volume systems created in high energy nuclear or particle collisions such QGP bags could appear like any other metastable states in statistical mechanics, since in this case the statistical suppression is just a few orders of magnitude and not of the order of the Avogadro number. Moreover, at the pre-equilibrated stage of high energy collision nothing can actually prevent their appearance. Then, if such QGP bags can be created there must be a reason which prevents their direct experimental detection. As we will show here there is an inherent property of the strongly interacting matter equation of state (EoS) which prevents their appearance inside of the hadronic phase even in finite systems. The same property is also responsible for the instability of large or heavy strangelets.

The second conceptual problem is rooted in a huge deficit of the number of observed hadronic resonances [15] with masses above 2.5 GeV predicted by the SBM and used, so far, by all other subsequent models discussed above. Thus, there is a paradox situation with the Hagedorn mass spectrum: it was predicted for heavy hadrons which nowadays must be regarded as QGP bags, but it can be experimentally established up to hadronic masses of about 2.3 GeV [15], whereas the recent review of Particle Data Group contains very few heavier hadronic resonances comparing to the SBM expectations. Moreover, the best description of particle yields observed in a very wide range of collision energies of heavy ions is achieved by the statistical model which incorporates s all hadronic resonances not heavier than 2.3 GeV [8]. Thus, it looks like heavier hadronic species, except for the long living ones, are simply absent in the experiments [16]. Of

course, one could argue that heavy hadronic resonances cannot be established experimentally because both their large width and very large number of decay channels lead to great difficulties in their identification, but the point is that, except for the recent efforts [16], the influence of large width of heavy resonances on their EoS properties and the corresponding experimental consequences were rather not studied in full.

Therefore, here we would like to introduce the finite and medium dependent width of QGP bags into the statistical model, study its impact on the pressure of system at zero baryonic density and show that the *subthreshold suppression of the QGP bags* of this finite width model (FWM) resolves both the conceptual problems discussed above. Our aim is to make a firm bridge between the statistical description of QGP, the LQCD results and the most general properties of hadronic mass spectrum as seen in experiments at high energies.

II. BASIC INGREDIENTS OF THE FWM

The most convenient way to study the phase structure of any statistical model similar to the SBM, GBM or the QGP bags with surface tension model (QGBSTM) [10] implies to use the isobaric partition [7, 10, 14] and find its rightmost singularities. Hence, after the Laplace transform the FWM grand canonical partition $Z(V,T)$ generates the following isobaric partition:

$$\hat{Z}(s,T) \equiv \int_0^\infty dV \exp(-sV)\ Z(V,T) = \frac{1}{[s - F(s,T)]}\,, \quad (1)$$

where the function $F(s,T)$ includes the discrete F_H and continuous F_Q mass-volume spectrum of the bags

$$F(s,T) \ \equiv\ F_H(s,T) + F_Q(s,T) = \sum_{j=1}^{n} g_j e^{-v_j s} \phi(T, m_j) + \int_{V_0}^\infty dv \int_{M_0}^\infty dm\ \rho(m,v) \exp(-sv) \phi(T,m)\ . \quad (2)$$

The bag density of mass m_k, eigen volume v_k and degeneracy factor g_k is given by $\phi_k(T) \equiv g_k\ \phi(T, m_k)$ with

$$\phi_k(T) \equiv \frac{g_k}{2\pi^2} \int_0^\infty p^2 dp\ e^{-\frac{(p^2\ +\ m_k^2)^{\frac{1}{2}}}{T}} = g_k \frac{m_k^2 T}{2\pi^2}\ K_2\left(\tfrac{m_k}{T}\right) .$$

The mass-volume spectrum $\rho(m,v)$ generalizes the exponential mass spectrum introduced by Hagedorn [1]. As in the GBM and QGBSTM, the FWM bags are assumed to have the hard core repulsion of the Van der Waals type generating the suppression factor proportional to the exponential of bag proper volume $\exp(-sv)$. The first term of Eq. (2), F_H, represents the contribution of a finite number of low-lying hadron states up to mass $M_0 \approx 2$ GeV [17]. F_H has no s-singularities at any temperature T and generates a simple pole (1) that describes a hadronic phase, whereas we will prove that the mass-volume spectrum of the bags $F_Q(s,T)$ leads to an essential singularity $s_Q^*(T) \equiv p_Q(T)/T$ which defines the QGP pressure $p_Q(T)$ at zero baryonic densities [7, 10, 17]. Any singularity s^* of $\hat{Z}(s,T)$ (1) is defined by the equation $s^*(T) \ =\ F(s^*,T)$ [7, 10].

Here we use the simplest parameterization of the spectrum $\rho(m,v)$ to demonstrate the idea. Additional physical justification of the FWM along with the analysis of the FWM relation to the Regge trajectories of heavy QGP bags and the corresponding experimental consequences can be found in Refs. [18] and [19], respectively. We, however, stress that the requirements discussed in the introduction do not leave us too much freedom to construct such a spectrum. Thus, to have a firm bridge with the most general experimental and theoretical findings of particle phenomenology it is necessary to assume that the continuous hadronic mass spectrum has a Hagedorn like form

$$\rho(m,v) = \frac{\rho_1(v)\ N_\Gamma}{\Gamma(v)\ m^{a+\frac{3}{2}}} \exp\left[\frac{m}{T_H} - \frac{(m-Bv)^2}{2\Gamma^2(v)}\right], \quad (3)$$

$$\rho_1(v) = f(T)\, v^{-b}\ \exp\left[-\frac{\sigma(T)}{T}\, v^\kappa\right]. \quad (4)$$

Also this spectrum has the Gaussian attenuation around the bag mass Bv determined by the volume dependent Gaussian width $\Gamma(v)$ or width hereafter. We will distinguish it from the true width defined as $\Gamma_R = \alpha\, \Gamma(v)$ ($\alpha \equiv 2\sqrt{2\ln 2}\,$).

In practice for narrow resonances there used two mass distributions, the Breit-Wigner and the Gaussian ones. As will be shown below the Gaussian dependence is of a crucial importance for the FWM because the Breit-Wigner attenuation leads to a divergency of the partition function. This is quite different from the early attempts to consider the width of QGP bags in [16].

The normalization factor in (3) is defined to obey the condition

$$N_\Gamma^{-1} \ =\ \int_{M_0}^\infty \frac{dm}{\Gamma(v)} \exp\left[-\frac{(m-Bv)^2}{2\Gamma^2(v)}\right]. \quad (5)$$

It is important that the volume spectrum in (4) contains the surface free energy ($\kappa\ =\ 2/3$) with the T-dependent surface tension which is parameterized by $\sigma(T) = \sigma_0 \cdot \left[\frac{T_c - T}{T_c}\right]^{2k+1}$ ($k\ =\ 0, 1, 2, ...$) [10, 20], where $\sigma_0 > 0$ could, nevertheless, be a smooth function of temperature. In [10] it is shown that such a parameterization of the bag surface tension is of crucial importance to generate the QCD tricritical endpoint. For T not above the tricritical temperature T_c this form of $\sigma(T)$ is justified by the usual cluster models like the Fisher droplet model [21] and SMM [22, 23], whereas the general T dependence can be analytically derived from the surface partitions of the Hills and Dales model [20]. The important consequences of such a surface tension and a discussion of the curvature free energy absence in (4) can be found in [10, 24].

An attempt of Ref. [17] to derive the bag pressure [4] within the GBM is based on a complicated mathematical construct, but does not explain an underlying physical reason for the mass-volume spectrum of bags suggested in [17]. In contrast to [17], the spectrum (3) (and (4)) is simple, but general and adequate for the medium dependence of both the width $\Gamma(v)$ and the bag's mass density B. It clearly reflects the fact that the QGP bags are similar to the ordinary quasiparticles with the medium dependent characteristics (life-time, most probable values of mass and volume). Now we are ready to derive the pressure of an infinite bag for two dependencies: the volume independent width $\Gamma(v) = \Gamma_0$ and the volume dependent width as $\Gamma(v) = \Gamma_1 \equiv \gamma v^{\frac{1}{2}}$.

III. ANALYSIS OF THE FWM SPECTRUM

First we note that for large bag volumes ($v \gg M_0/B > 0$) the factor (5) can be found as $N_\Gamma \approx 1/\sqrt{2\pi}$. Similarly, one can show that for heavy free bags ($m \gg BV_0$, $V_0 \approx 1$ fm^3 [17], ignoring the hard core repulsion and thermostat)

$$\rho(m) \equiv \int\limits_{V_0}^{\infty} dv\, \rho(m,v) \approx \frac{\rho_1(\frac{m}{B})}{B\ m^{a+\frac{3}{2}}} \exp\left[\frac{m}{T_H}\right], \quad (6)$$

i.e. the spectrum (3) integrated over the bag volume has a Hagedorn form modified by the surface free energy. It results from the fact that for heavy bags the Gaussian in (3) acts as a Dirac δ-function for either choice of Γ_0 or Γ_1. Thus, the Hagedorn form of (6) receives a clear physical meaning and gives an additional argument in favor of the FWM. Also it gives an upper bound for the volume dependence of $\Gamma(v)$: the Hagedorn-like mass spectrum (6) can be derived, if for large v the width Γ increases slower than $v^{(1-\kappa/2)} = v^{2/3}$.

Similarly to Eq. (6), one can estimate the width of heavy free bags averaged over their volumes and get $\overline{\Gamma(v)} \approx \Gamma(m/B)$. Thus, with choosing $\Gamma(v) = \Gamma_1(v)$ the mass spectrum of heavy free QGP bags is found to be the Hagedorn-like one and heavy resonances develop the large mean width $\Gamma_1(m/B) = \gamma\sqrt{m/B}$. Hence, they are hard to be observed. Applying these arguments to the strangelets, we conclude that, if their mean volume is a few cubic fermis or larger, they should survive for a very short time, which is in line with the results of [25].

Note also that such a mean width is essentially different from both the linear mass dependence of string models [26] and from an exponential form of the nonlocal field theoretical models [27].

Next we calculate $F_Q(s,T)$ (2) for the spectrum (3) performing the mass integration. There are two distinct options depending on the sign of the most probable mass:

$$\langle m\rangle \equiv Bv + \Gamma^2(v)\beta\,, \quad \text{with} \quad \beta \equiv T_H^{-1} - T^{-1}\,. \quad (7)$$

If $\boldsymbol{\langle m\rangle > 0}$ for $v \gg V_0$, one can use the saddle point method for mass integration to find the function $F_Q(s,T)$

$$F_Q^+(s,T) \approx \left[\frac{T}{2\pi}\right]^{\frac{3}{2}} \int\limits_{V_0}^{\infty} dv\ \frac{\rho_1(v)}{\langle m\rangle^a}\ \exp\left[\frac{(p^+ - sT)v}{T}\right] \quad (8)$$

and the pressure of large bags $p^+ \equiv T\left[\beta B + \frac{\Gamma^2(v)}{2v}\beta^2\right]$. To get (8) one has to employ in (2) an asymptotics of the K_2-function $\phi(T,m) \simeq (mT/2\pi)^{3/2}\exp(-m/T)$ for $m \gg T$, collect all m-dependent terms in exponential, get a full square for $(m - \langle m\rangle)$ and perform the Gaussian integration.

Since for $s < s_Q^*(T) \equiv p^+(v \to \infty)/T$ the integral (8) diverges on its upper limit, the partition (1) has an essential singularity corresponding to the QGP pressure inside of an infinite bag. It allows one to conclude the width Γ cannot increase faster than $v^{1/2}$ for $v \to \infty$, otherwise $p^+(v \to \infty) \to \infty$ and $F_Q^+(s,T)$ diverges for any s. Thus, for $\boldsymbol{\langle m\rangle > 0}$ the phase structure of the FWM with $\Gamma(v) \neq 0$ is similar to the QGBSTM [10].

The bag spectrum $F_Q^+(s,T)$ (8) is of general nature and, unlike the suggestion of [17], has a transparent physical origin. One can also see that two general sources of the bulk part of bag free energy

$$-p^+ v = -T\left[\beta\langle m\rangle - \tfrac{1}{2}\Gamma^2(v)\beta^2\right] \quad (9)$$

are the bag most probable mass and its width. Different T dependent functions $\langle m\rangle$ and $\Gamma^2(v)$ lead to different EoS.

If instead of the Gaussian width parameterization in (3) we used the Breit-Wigner one, then we would not be able to derive the continuous spectrum $F_Q^+(s,T)$ (8) and the corresponding bag pressure for any nonvanishing bag width $\Gamma(v)$. Indeed, for $T > T_H$ the mass integrals in $F_Q(s,T)$ would diverge like in SBM, unless the Breit-Wigner mass attenuation has a zero width or an exponentially increasing width $\Gamma \sim \exp[m/T_H]$ [16]. The former does not resolve the both of the GBM conceptual problems, whereas the latter corresponds to a very specific ansatz for the resonance width which is in contradiction with the FWM assumptions.

It is possible to use the spectrum (8) not only for infinite system volume but for finite volumes $V \gg V_0$ as well. In this case the upper limit of integration should be replaced by finite V (see Ref. [9] for details). It changes the singularities of partition (1) to a set of simple poles $s_n^*(T)$ in the complex s-plane which are defined by the same equation as for $V \to \infty$. Similarly to the finite V solution of the GBM [9], it can be shown that for finite T the FWM simple poles may have a small positive or even negative real part which would lead to a non-negligible contribution of the QGP bags into the spectrum $F(s,T)$ (2). Thus, if the spectrum (8) was the only volume spectrum of the QGP bags, then there would exist the non-negligible probability of finding heavy QGP bags ($m \gg M_0$) in finite systems at $T \ll T_H$. Therefore, using the results of the finite volume GBM and SMM, we conclude that the spectrum (8) itself cannot explain

the absence of the QGP bags at $T \ll T_H$ and, hence, an alternative explanation of this fact is required.

Such an explanation corresponds to the negative values of $\langle \boldsymbol{m} \rangle \leq \mathbf{0}$ for $v \gg V_0$. From (7) one can see that for the volume dependent width $\Gamma(v) = \Gamma_1(v)$ the most probable mass $\langle m \rangle$ inevitably becomes negative at low T, if $0 < B < \infty$. Using the asymptotics of the K_2-function for large and small values of $\frac{m}{T}$ one can show that at low T the maximum of the Gaussian mass distribution is located at $\langle m \rangle \leq 0$. Hence only the tail of the Gaussian mass distribution close to M_0 contributes to $F_Q(s,T)$. By the steepest descent method and with the K_2-asymptotic form for $M_0 T^{-1} \gg 1$ one gets

$$F_Q^-(s,T) \approx \left[\frac{T}{2\pi}\right]^{\frac{3}{2}} \int\limits_{V_0}^{\infty} dv \frac{\rho_1(v) N_\Gamma\, \Gamma(v)\, \exp\left[\frac{(p^- - sT)v}{T}\right]}{M_0^a\, [M_0 - \langle m \rangle + a\, \Gamma^2(v)/M_0]} \quad (10)$$

with the analytic form for the QGP bag pressure

$$p^-\big|_{v \gg V_0} = \frac{T}{v}\left[\beta M_0 - \frac{(M_0 - Bv)^2}{2\,\Gamma^2(v)}\right]. \quad (11)$$

We would like to stress the last result requires $B > 0$ and cannot be generated by a weaker v-dependence than $\Gamma(v) = \Gamma_1(v)$. Indeed, if $B < 0$, then the normalization factor (5) would not be $1/\sqrt{2\pi}$, but changes to $N_\Gamma \approx [M_0 - \langle m \rangle]\, \Gamma^{-1}(v) \exp\left[\frac{(M_0 - Bv)^2}{2\,\Gamma^2(v)}\right]$ and, thus, it would cancel the leading term in pressure (11). Note that the inequality $\langle \boldsymbol{m} \rangle \leq \mathbf{0}$ for all $v \gg V_0$ with $B > 0$ and finite $p^-(v \to \infty)$ is valid for $\Gamma(v) = \Gamma_1(v)$ only. The negative value of $\langle m \rangle$ is an indicator of a different physical situation comparing to $\langle m \rangle > 0$, but has no physical meaning since $\langle m \rangle \leq 0$ does not enter the main physical observable p^-.

The new outcome of this case with $B > 0$ is that for $T < T_H$ the spectrum (10) contains the lightest QGP bags having the smallest volume since every term in the pressure (11) is negative. The finite volume of the system is no longer important because only the smallest bags survive in (10). Moreover, if such bags are created, they would have the masses of about M_0 and the widths of about $\Gamma_1(V_0)$, and, hence, they would hardly be distinguishable from the usual low-mass hadrons. Thus, the situation $\langle \boldsymbol{m} \rangle \leq \mathbf{0}$ with $B > 0$ leads to the *subthreshold suppression of the QGP bags* at low temperatures, since their most likely mass is below the mass threshold M_0 of the spectrum $F_Q(s,T)$. Note that such an effect cannot be derived within any of the GBM-kind models proposed earlier.

The results received give us a unique opportunity to make a bridge between the particle phenomenology, some experimental facts and LQCD conclusions. For instance, if the most probable mass of the QGP bags is known along with the QGP pressure, one can estimate the width of these bags directly from Eqs. (9) and (11). The FWM pressure depends on two functions, therefore, in order to find them it is necessary to know the form of the QGP pressure somewhere in the hadronic phase. Unfortunately, the present LQCD data do not provide us with such a detailed information and, hence, at the moment some additional assumptions are inevitable. To demonstrate the new possibilities of FWM let us consider several examples of the QGP EoS and relate them to the above results.

First, we study the possibility to get the MIT bag model pressure $p_{bag} \equiv \sigma T^4 - B_{bag}$ [4] by the stable QGP bags, i.e. $\Gamma(v) \equiv 0$. Equating the pressures p^+ and p_{bag}, one finds that T_H must be related to a bag constant as $B_{bag} \equiv \sigma T_H^4$. Then the mass density of such bags $\frac{\langle m \rangle}{v} \equiv B = \sigma T_H (T + T_H)(T^2 + T_H^2)$ is always positive. Thus, the MIT bag model EoS can be easily obtained by the FWM approach, but, as discussed earlier, such bags should have been observed.

Secondly, we consider the stable bags, $\Gamma(v) \equiv 0$, but without the Hagedorn spectrum, i.e. $T_H \to \infty$. Matching $p^+ = -B$ and p_{bag}, we find that at low temperatures the bag mass density $\frac{\langle m \rangle}{v} = B$ is positive, whereas for high T the mass density cannot be positive and, hence, one cannot reproduce p_{bag} as $B \leq 0$ and the resulting pressure is not p^- (11), but a zero, as seen from (10), (11) and N_Γ expression for the limit $\Gamma(v) \to 0$. One can try to reproduce p_{bag} with the finite T dependent width $\Gamma(v) = 2\,\sigma T^5 v$ for $T_H \to \infty$. Then one can get p_{bag} from p^+, but only for low temperatures obeying the inequality $\frac{\langle m \rangle}{v} = B_{bag} - 2\,\sigma T^4 > 0$. Thus, these two examples teach us that without the Hagedorn mass spectrum one cannot get the MIT bag model pressure.

It is also possible to reproduce an alternative QGP EoS $p_a = \sigma T^4 - A_1 T + A_0$ ($A_1 > 0$, $A_0 \geq 0$) [28] even for $\Gamma(v) \equiv 0$. The linear T-dependence in the QGP pressure, which clearly has nonperturbative nature, is seen [18] both in old [29, 30] and fresh [31] LQCD data. Choosing T_H to be a positive solution of equation $A_0 = A_1 T_H - \sigma T_H^4$, one obtains p_a from p^+ for the mass density of bag $\frac{\langle m \rangle}{v} \equiv B = \sigma T T_H (2\,T^2 + T T_H + T_H^2) - A_0$. If $A_0 = 0$ (found in [18, 28] from the LQCD data [29, 30, 31]), FWM is able to reproduce p_a for any T, whereas for $A_0 > 0$ it works for temperatures obeying $B > 0$.

Also the model with the linear T-dependent pressure p_a and $A_0 = 0$ allows us to estimate roughly the width $\Gamma_1(V_0)$ in a FWM. Matching p_a with $p^-(v \to \infty) = -T \frac{B^2}{2\,\gamma^2}$ one can determine B/γ ratio for $T \leq c_\pm T_H$ ($0 < c_\pm < 1$). For $T = 0$ one finds $A_1 = \frac{B_0^2}{2\,\gamma_0^2}$, i.e. the FWM naturally explains the linear T-dependent term in QGP pressure for the nonvanishing bag width coefficient γ_0 at $T = 0$.

Putting p^+ and $p^-(v \to \infty)$ equal and solving for γ^2, one can find the switching value of the width coefficient $\gamma_\pm^2 = -\frac{B}{\beta}$ for the switching temperature $T_\pm = c_\pm T_H$. From this result one can show that $0 < c_\pm < 1$ due to the inequalities $B > 0$ and $\gamma_\pm^2 > 0$.

Then matching p_a and p^+, one obtains the width coefficient

$$\gamma^2 = 2\,\beta^{-1}[\sigma T_H T (T^2 + T T_H + T_H^2) - B(T)] \quad (12)$$

for $T \geq c_{\pm} T_H$. Obviously, if $(T - T_H)$ is an exact divisor of the difference in (12), then $\gamma^2 > 0$ for all temperatures in the range $c_{\pm} T_H \leq T \leq T_H$. The simplest possibility to obey such a requirement is to assume that $B(T) = \sigma T_H^2 (T^2 + T T_H + T_H^2)$ for any T. Then one gets $\gamma_0^2 = B_0^2/(2A_1) = T_H B_0/2 = \sigma T_H^5/2$ for $T = 0$ and $\gamma^2 = 2TB(T)$ for $T \geq T_H$. Taking the constants in T_c units ($T_c \approx 200$ MeV), we obtain the true width for the SU(3) color group with two flavors [30] as $\Gamma_R(V_0, T = 0) \approx 1.22\, V_0^{\frac{1}{2}}\, T_c^{\frac{5}{2}} \alpha \approx 587$ MeV and $\Gamma_R(V_0, T = T_H) = \sqrt{12}\, \Gamma_R(V_0, T = 0) \approx 2034$ MeV. This estimate clearly demonstrates us that there is no way to detect the decays of such shortly living QGP bags in the laboratory. A detailed analysis of these findings and their sensitivity to different LQCD data is presented in [18].

IV. CONCLUSIONS

Here we develop the novel statistical approach to study the QGP bags with medium dependent finite width. The FWM is based on the Hagedorn-like mass spectrum of bags modified by the surface free energy of bags and by the bag width. We show that the volume dependent width of the QGP bags $\Gamma(v) = \gamma\, v^{\frac{1}{2}}$ leads to the Hagedorn mass spectrum of free heavy bags. Such a behavior of a width allows us to explain a deficit of heavy hadronic resonances in the experimentally observed mass spectrum. Under the plausible assumptions we derive the general form for the bag pressure p^+ which accounts for the effect of finite width in the EoS. We argue that the obtained spectrum itself cannot explain the absence of directly observable QGP bags in the high energy nuclear and particle collisions. Then we find out a new possibility to "hide" the heavy QGP bags for $T \ll T_H$ by their *subthreshold suppression*. The latter occurs due to the fact that at low T the most probable mass of heavy bags $\langle m \rangle \leq 0$ and, thus, is below the lower cut-off M_0 of the continuous mass spectrum. Hence only the lightest bags of mass about M_0 and of smallest volume V_0 may contribute into the resulting spectrum, but such QGP bags will be indistinguishable from the low-lying hadronic resonances. We demonstrate how the FWM can reproduce a few EoS on the QGP market and corroborate that the low T pressure p^- reproduces properly some nonperturbative features revealed by LQCD. In principle, the FWM allows one to extract the QGP pressure from the LQCD pressure for hadronic phase, if the contribution of the discrete part of hadronic mass-volume spectrum is also known. The generalization to non-zero baryonic densities is straightforward by assuming the dependence of the model functions B and $\Gamma_1(v)$ on the baryonic chemical potential. Our estimate of the volume dependent width looks pretty encouraging for heavy ion phenomenology. A detailed discussion of the FWM experimental consequences will be presented elsewhere [18, 19]. To simultaneously determine the most probable mass of the QGP bags and their width, it would be nice to study the metastable branch of the QGP EoS at low T with the LQCD and compare its results with the FDM.

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